Sag Equation for Coatings on Rotating Substrates

H. L. FRISCH[1] **and C. SCOPAZZI**[2,*]

[1]Department of Chemistry, State University of New York, Albany, New York 12222, and [2]E.I. DuPont de Nemours and Company, Marshall R&D Laboratory, 3500 Grays Ferry Avenue, Philadelphia, Pennsylvania 19146

SYNOPSIS

We develop the sag length equation for coatings on rotating substrates, neglecting Coriolis force effects. The equation is integrated analytically for both Newtonian and some cases of non-Newtonian flow. For rotating substrates, the sag in one direction is compensated for by sag in the opposite direction, and unlike the nonrotating case, the sag is not proportional to time.

INTRODUCTION

The appearance or aesthetics of an automobile is an important aspect in the marketing of the vehicle. Most current automotive coatings are multilayered with a primed substrate on which is applied a color coat with a nonpigmented clear coat as the outermost coating (Fig. 1). Both color and clear coats are based on cross-linking chemistries that require stoving at elevated temperatures. It is the smoothness of the clear coat that is very important in determining the overall aesthetics of the applied coating. This smoothness is quantified by the measurement of the distinctness of image (DOI),[1] which is sensitive to distortions or roughness of small-length scales. Desired levels of DOI are typically in excess of 90. A problem in the utilization of color-coat/clear-coat coatings on automobiles is the difference in the DOI of horizontal and vertical substrates or parts of the vehicle. In general, the horizontal DOI is in excess of 90, but the vertical DOI can be as low as 50 depending on the pigmentation of the color coat and thickness of the clear coat. The increased roughness of the clear coat on vertical substrates has been ascribed to sag of the clear coat that occurs at elevated temperatures[2,3] and the concomitant telegraphing of the rough color coat.[4] A good correlation of vertical DOI to calculate sag lengths of the clear coat has been made.[3] The sag length is calculated utilizing the well-known equation for sag of Newtonian liquids[5] and through a knowledge of measured clear-coat viscosity at elevated temperatures. As shown in Figure 2, one needs to keep the sag length less than about 0.1 cm in order to achieve vertical DOIs in excess of 80, which is generally considered to be acceptable.

The reduction of sag and, hence, the vertical DOI performance of the clear coat can be improved by the addition of rheology control agents such as silica. However, the increase in viscosity needed to improve vertical DOI performance can have a negative impact on the flow and leveling of the clear coat on horizontal surfaces. Thus, the improvement in the DOI of the clear coat on vertical surfaces is often at the expense of the smoothness and DOI of the coating on horizontal surfaces. An approach around the issue of differential DOI performance of clear coats on horizontal and vertical surfaces and one pioneered by the Mazda Motor Corporation is rotation of the car body on the horizontal axis while baking. This minimizes the degree of sag and also makes it the same for all surfaces, leading to a uniform, overall appearance of high DOI. The objective of this communication is to develop an equation for sag that covers this type of situation.

THEORY AND DISCUSSION

The following development neglects effects arising from Coriolis forces. This neglect is justified with

* To whom correspondence should be addressed.

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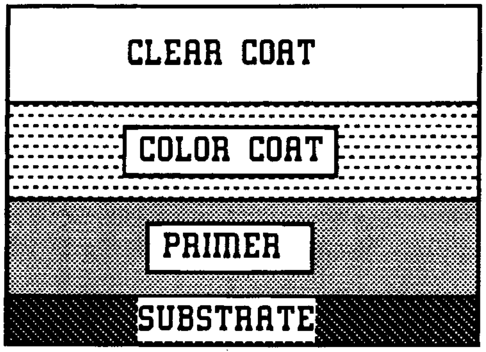

Figure 1 Automotive multilayered coatings.

the low angular velocities employed in these applications. In deriving the sag equation for rotating substrates, we follow the procedure outlined in Ref. 5 for the sag equation for nonrotating substrates. We visualize a coating of thickness H applied to a vertical substrate (Fig. 3) that rotates at a frequency ν (cycles/s) about the y axis. The coating can be thought to consist of thin layers each of thickness dx. Consider one of these layers at a depth x and of unit surface area. The shear stress on this layer is

$$\tau_x(t) = \rho g x \cos(2\pi\nu t) \tag{1}$$

where g is the acceleration of gravity (980 cm/s^2). The tendency for this layer to sag arises from the presence of this shear stress and is offset by the viscous resistance of the coating. In general, one can relate the shear stress to the viscosity and shear rate via the relationship

$$\tau = K(D)^n \tag{2}$$

where D is the shear rate ($= dv/dx$); v, the velocity; K, the consistency; and n, the power law index. $n = 1$ for Newtonian fluids, whereas for non-Newtonian fluids, $n < 1$ (shear thinning) and $n > 1$ (dilatant). Equating eqs. (1) and (2)

$$dv/dx = [\rho g \cos(2\pi\nu t)/K]^{1/n} x^{1/n} \tag{3}$$

Integrating over x over the limits 0 to H gives

$$\begin{aligned} v(H, t) &= [\rho g/K]^{1/n} \cos^{1/n}(2\pi\nu t) \int_0^H x^{1/n}\, dx \\ &= [\rho g/K]^{1/n} \cos^{1/n}(2\pi\nu t) \\ &\quad \times [H^{1/n+1}/(1/n + 1)] \end{aligned} \tag{4}$$

The sag length S is given by

$$S = \int_0^t v(H, t)\, dt \tag{5}$$

To calculate the sag length S we look at three cases—Newtonian for which $n = 1$, the non-Newtonian or shear thinning case for which $n < 1$, and the dilatant case for which $n > 1$.

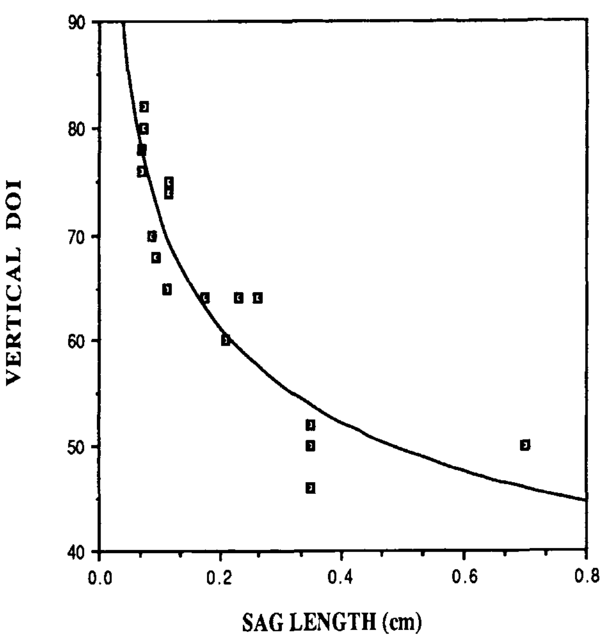

Figure 2 Vertical DOI of clearcoat versus calculated sag length.

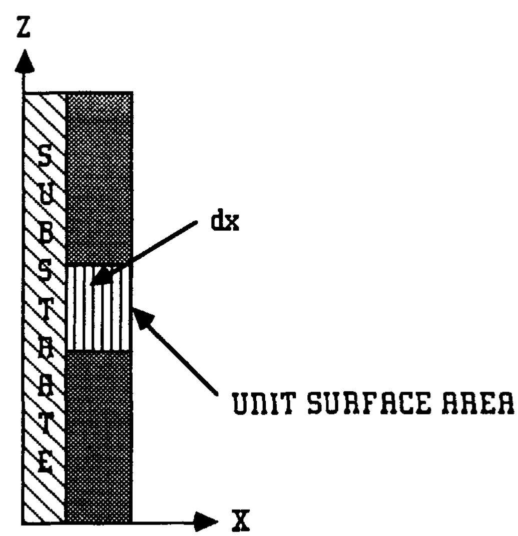

Figure 3 Initial geometry of coating on a vertical substrate.

Newtonian

Integrating eq. (5) over the limits 0 to t and setting $n = 1$ gives

$$S = [\rho g H^2/(4\pi\nu\eta)]\sin(2\pi\nu t) \qquad (6)$$

Equation (6) indicates that the sag length varies sinusoidally with time and its maximum is inversely proportional to the frequency of rotation as well as to the viscosity. This is evident in Figure 4 where S is plotted versus the frequency of rotation in rpm. We have used a value of 50 microns for the coating thickness H and have set the viscosity, η, equal to 10 poise. These values are typical of automotive (OEM) clear coats based on acrylic polymers with no rheology control additives at temperatures encountered early in the stoving process.[3] A positive value of S in Figure 4 implies sag in the direction of the original configuration, whereas a negative value of S indicates sag in the opposite direction. The implication of eq. (6) is that sag control through substrate rotation arises from sag in the first 180° cycle being exactly compensated for by sag in the opposite direction in the subsequent 180–360° cycle and is thus independent of time. This is in contrast to nonrotating substrates where the degree of sag is cumulative (in time). The maximum sag length versus rpm is plotted in Figure 5 for two values of the viscosity: 10 and 20 poise. For a viscosity of 10 poise, the maximum value of the sag is less than 0.1 cm for rotations in excess of 0.12 rpm. At the higher viscosity of 20 poise, maximum sag lengths of 0.1 cm or less are achieved at lower rpm's of 0.06 and higher.

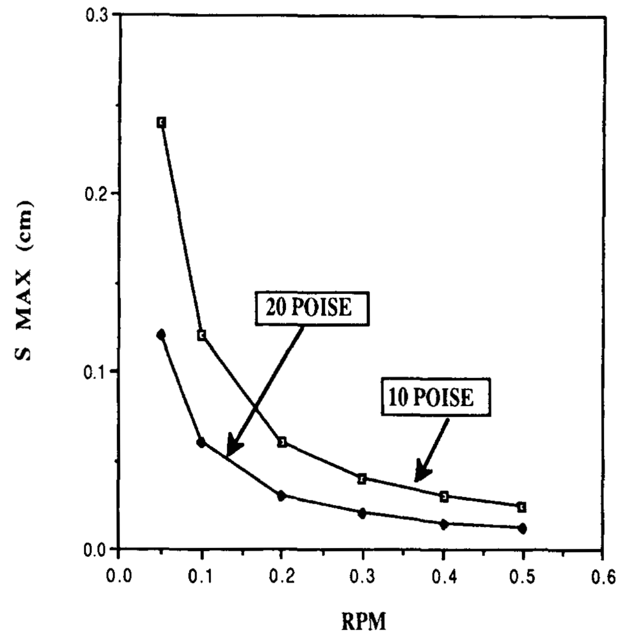

Figure 5 Maximum calculated sag length versus RPM for 10 and 20 poise viscosity. Coating thickness is 50 microns.

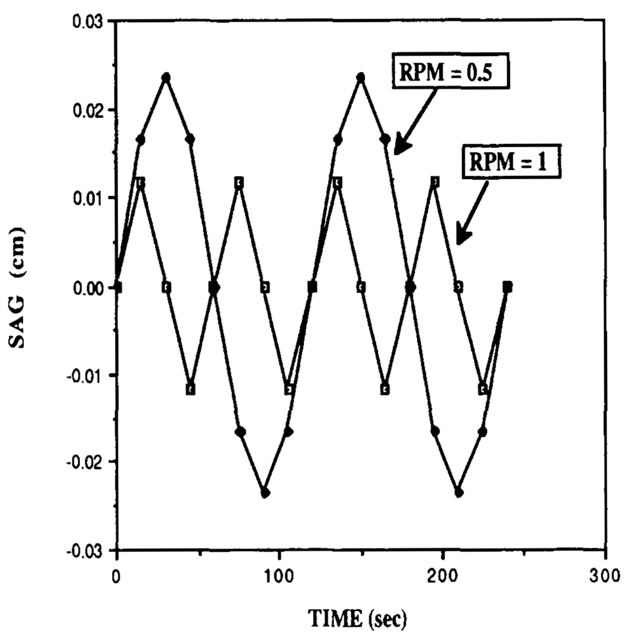

Figure 4 Calculated sag length of clear coat versus time for RPM's indicated. Viscosity is 10 poise and coating thickness is 50 microns.

Non-Newtonian

In general, for $n = 1$,

$$S = [\rho g/K]^{1/n}[H^{1/(n+1)}/(1/n + 1)] \times \int_0^t \cos^{1/n}(2\pi\nu t)\, dt \qquad (7)$$

and the time integral

$$\int_0^t \cos^{1/n}(2\pi\nu t)\, dt = 1/(2\pi\nu)\int_0^{2\pi\nu t} \cos^{1/n}\alpha d\alpha \qquad (8)$$

can be evaluated only by numerical integration. Analytical formulas can be found for a number of special, but technically important cases, using indefinite integrals.[6] For the shear thinning (pseudoplastic) case in which n is the reciprocal of an integer, we have (setting $2\pi\nu t = \theta$) for $n = 1/2m$, where m is an integer[6]:

$$S = (2\pi\nu)^{-1}(\rho g/K)^{2m}(H^{2m+1}/(2m+1)) \times [(2)^{-2m}\binom{2m}{m}\theta + (2)^{1-2m}\sum_{j=0}^{m-1}\binom{2m}{j} \times \sin(2m-2j)\theta/(2m-2j)] \quad (9)$$

and for $n = 1/(2m+1)$, where m is an integer:

$$S = (2\pi\nu)^{-1}(\rho g/K)^{2m+1}(H^{2m+2}/(2m+2)) \times [(2)^{-2m}\sum_{j=1}^{m}\binom{2m+1}{j}\sin(2m-2j+1)\theta/(2m-2j+1) \quad (10)$$

$\binom{a}{b}$ is the binomial coefficient and is equal to $a!/[(a-b)!b!]$. Note that when m is set equal to zero in eq. (10) we recover the Newtonian result eq. (6) as we should.

It is well known in the art that the introduction of shear sensitivity reduces the degree of sag. This arises from the fact that the sag length S is proportional to $H^{1+1/n}/K^{1/n}$ [see eq. (9) or (10)]. For $n = 1/2$, the dependence of S on coating thickness H goes as the third power versus the square for Newtonian systems. Since H is very small, typically less than 50 microns, S is commensurately smaller for non-Newtonian coatings. The added feature of a rotating substrate as shown for the Newtonian case is to reduce the maximum value of S further by a factor of $2\pi\nu$ [eqs. (9) and (10)].

The sag length S in the shear thickening (dilatant), $n > 1$, case can be expressed only in terms of higher transcendental functions. For the case $n = 2$ (Ref. 6),

$$S = (\rho g/K)^{1/2}(2/3)H^{3/2}(\pi\nu)^{-1}\{(2)^{0.5}E[\gamma, 1/(2)^{0.5}] - 1/(2)^{0.5}F[\gamma, 1/(2)^{0.5}]\} \quad (11)$$

with $\gamma = \arcsin[(2)^{0.5}\cos \pi\nu t]$, $0 < 2\pi\nu t < \pi/2$, and E and F are the first and second elliptic integrals of the indicated arguments.

CONCLUSIONS

Equations for the sag length for coatings on rotating substrates have been derived for both Newtonian and non-Newtonian cases. The principal idea is that sag in one direction is fully compensated for by sag in the reverse direction and, in contrast to the case for nonrotating substrates, sag is not cumulative in time. The sag length depends inversely on the rate of rotation, and for a Newtonian viscosity of 10 poise, typical of current automotive clear coats early in the stoving process, an rpm of 0.12 or more is required to keep sag lengths less than 0.1 cm, beyond which a negative effect on clear-coat smoothness would be expected.

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